

## **CHAPTER - 7**

# **TIME-VARYING ELECTROMAGNETIC FIELDS**

# " CHAPTER-7 TIME-VARYING EM FIELDS "

Exercise 7.1  $L = 2\text{ m}$   $\vec{B} = 12.5 \vec{a}_2 \text{ mT}$   $\omega = 188 \text{ rad/s}$

$$e_{ba} = \frac{1}{2} B \omega L^2 = 4.7 \text{ V} \quad I_{ba} = e_{ba} / 2 = 2.35 \text{ A}$$

$$P_{\text{sup}} = 4.7 \times 2.35 = 11.05 \text{ W}$$

$$d\vec{l} = d\rho \vec{a}_\rho \Rightarrow \vec{F}_m = \int \vec{dl} \times \vec{B} = -IB \int d\rho \vec{a}_\rho = -58.75 \vec{a}_\phi \text{ mN}$$

$$\text{Thus, } \vec{F}_{\text{ext}} = 58.75 \vec{a}_\phi \text{ mN}$$

Exercise 7.2  $L = 1\text{ m}$ ,  $e_{ba} = \frac{1}{2} \times 12.5 \times 10^{-3} \times 188 \times 1^2 = 1.175 \text{ V}$

$$\text{For } R = 2\Omega, I_R = 0.5875 \text{ A}, I_{\text{strip}} = 293.75 \text{ mA}, P_{\text{sup}} = \frac{1.175^2}{2} = 0.69 \text{ W}$$

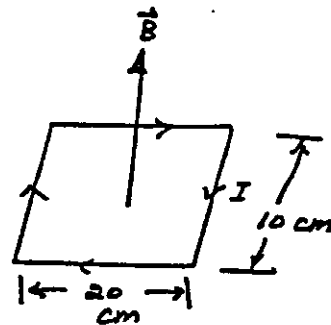
Exercise 7.3  $l = 20 + 10 + 20 + 10 = 60 \text{ cm}$

$$r = 1.2 \text{ mm}, A = \pi r^2 = 4.524 \times 10^{-6} \text{ m}^2$$

$$R = \frac{l}{\sigma A} = 371.51 \mu\Omega, \text{ where } \sigma = 3.57 \times 10^7 \text{ S/m}$$

$$|e| = N \frac{d\Phi}{dt} = (+1)(200 \times 10^4) \frac{dB}{dt}$$

$$= 200 \times 10^{-4} \times 40 = 0.8 \text{ V} \Rightarrow I = 2153.4 \text{ A as shown.}$$



Exercise 7.4

$$\vec{B} = 12.5 \vec{a}_2 \text{ mT}$$

$$d\vec{s} = \rho d\rho d\phi \vec{a}_2$$

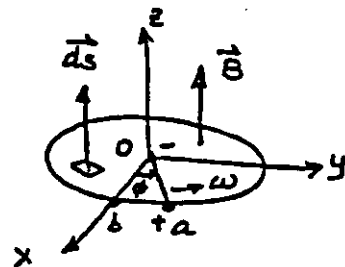
$$\Phi = \int_S \vec{B} \cdot d\vec{s} = \int_0^a \int_0^{2\pi} 12.5 \times 10^{-3} \rho d\rho \int_0^\phi d\phi$$

$$= 0.025\phi = 0.025\omega t$$

$$\omega = 188 \text{ rad/s}$$

$$e = -N \frac{d\Phi}{dt} = -0.025\omega = -4.7 \text{ V}$$

As the flux passing thro' loop is increasing with time, a must be at a higher potential than b. Thus,  $e_{ao} = 4.7 \text{ V}$



### Exercise 7.5

$$f = 200 \text{ kHz}$$

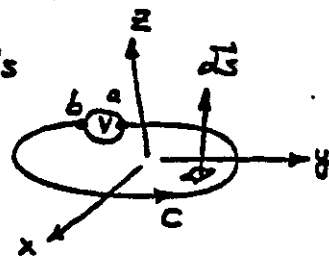
Let  $\vec{H} = 10 \cos \omega t \vec{a}_z$  where  $\omega = 2\pi f = 1.257 \times 10^6 \text{ rad/s}$

$$\vec{B} = \mu_0 \vec{H} \Rightarrow \frac{\partial \vec{B}}{\partial t} = -15.8 \sin \omega t \vec{a}_z$$

$$d\vec{s} = \rho d\rho d\phi \vec{a}_z$$

$$e_{ab} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} = 15.8 \sin \omega t \int_0^{0.1} \rho d\rho \int_0^{2\pi} d\phi$$

$$= 0.496 \sin \omega t \Rightarrow \text{Voltmeter reading; } \frac{0.496}{\sqrt{2}} = 0.35 \text{ V}$$



### Exercise 7.6

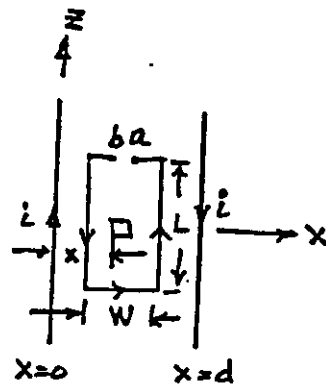
$$i = I_m \sin \omega t \text{ A}$$

$$\vec{B} = \frac{\mu_0 I_m \sin \omega t}{2\pi} \left[ \frac{1}{x} + \frac{1}{d-x} \right] \vec{a}_y \quad d\vec{s} = dx dz \vec{a}_y$$

$$\Phi = \int_S \vec{B} \cdot d\vec{s} = \frac{\mu_0 I_m \sin \omega t}{2\pi} \int_{\frac{d-w}{2}}^{\frac{d+w}{2}} \left( \frac{1}{x} + \frac{1}{d-x} \right) dx \int_0^L dz$$

$$= \frac{\mu_0 L I_m}{\pi} \sin \omega t \ln \left[ \frac{d+w}{d-w} \right]$$

$$e_{ab} = -N \frac{d\Phi}{dt} = -\frac{N \omega L \mu_0 I_m}{\pi} \cos \omega t \ln \left[ \frac{d+w}{d-w} \right]$$



### Exercise 7.7

$$i = 2 \sin 314 t \text{ A}$$

as per:  $\oint_C \vec{H} \cdot d\vec{l} = Ni \Rightarrow$

$$H_\phi = \frac{Ni}{2\pi\rho} \quad , \quad B_\phi = \frac{\mu Ni}{2\pi\rho}$$

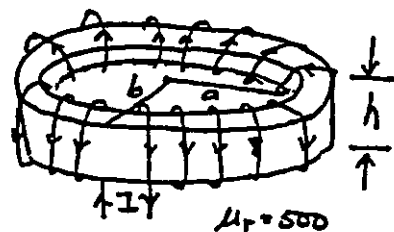
$$\Phi = \int_S \vec{B} \cdot d\vec{s} = \frac{\mu Ni}{2\pi} \int_a^b \frac{1}{\rho} d\rho \int_0^h dz$$

$$= \frac{\mu Ni h}{2\pi} \ln(b/a)$$

$$L = \frac{\lambda}{i} = \frac{N\Phi}{i} = \frac{\mu N^2 h}{2\pi} \ln(b/a) \cdot \text{Substitute values. } L = 44.63 \text{ mH}$$

$$i = 2 \sin 314 t, \text{ A} \quad , \quad e = -L \frac{di}{dt} = -44.63 \times 10^{-3} \times 2 \times 314 \cos 314 t$$

$$= -28.03 \cos 314 t, \text{ V}$$



$$a = 20 \text{ cm}, \quad b = 25 \text{ cm}$$

$$h = 5 \text{ cm}, \quad N = 200 \text{ Turns}$$

$$d\vec{s} = \rho d\rho dz \vec{a}_\phi$$

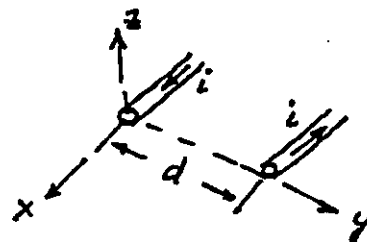
### Exercise 7.8

Assume  $d \gg a$ .  $\vec{e}$  current is distributed over the surface ( $\sigma \rightarrow$  very high)

$$B_z = \frac{\mu_0 I}{2\pi} \left[ \frac{1}{y} + \frac{1}{d-y} \right]$$

on a per-unit length basis:  $\vec{\Phi} = \frac{\mu_0 I}{2\pi} \int_a^{d-a} \left( \frac{1}{y} + \frac{1}{d-y} \right) dy = \frac{\mu_0 I}{\pi} \ln\left(\frac{d-a}{a}\right)$   
 $\approx \frac{\mu_0 I}{\pi} \ln\left(\frac{d}{a}\right) \text{ Wb/m}$

Thus,  $L = \frac{\mu_0}{\pi} \ln(d/a) \text{ H/m}$ .

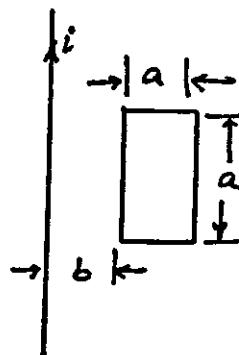


### Exercise 7.9

$$B_\phi = \frac{\mu_0 i}{2\pi r}$$

$$\vec{\Phi} = \frac{\mu_0 i}{2\pi} \int_b^{b+a} \frac{1}{r} dr \int_0^a dz = \frac{\mu_0 i}{2\pi} a \ln\left(\frac{b+a}{a}\right)$$

$$M = \frac{\mu_0}{2\pi} a \ln\left(\frac{b+a}{a}\right)$$



### Exercise 7.10

$$M = 16 \text{ mH}, L_1 = 20 \text{ mH}, L_2 = 80 \text{ mH}$$

$$M = k\sqrt{L_1 L_2} \Rightarrow k = \frac{16}{\sqrt{20 \times 80}} = 0.4$$

### Exercise 7.11

Refer to Fig. 7.19 [Parallel aiding]

$$v(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad (1)$$

$$v(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad (2)$$

From (1) and (2):  $\frac{di_2}{dt} = \frac{L_1 - M}{L_2 - M} \frac{di_1}{dt} \quad (3)$

Since  $i = i_1 + i_2$ ,  $\frac{di_2}{dt} = \frac{di}{dt} - \frac{di_1}{dt} \quad (4)$

From (3) and (4)

$$\frac{di}{dt} = \frac{L_1 + L_2 - 2M}{L_2 - M} \frac{di_1}{dt} \quad (5)$$

If  $L$  is the eqvt. inductance,  
 $v(t) = L \frac{di}{dt} \quad (6)$

From (1), (5) and (6)

$$L \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \text{or}$$

$$L \left[ \frac{L_1 + L_2 - 2M}{L_2 - M} \right] \frac{di_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{L_1 - M}{L_2 - M} \frac{di_1}{dt}$$

$$\text{Thus, } L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Similarly, you can show that

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \text{ for parallel-opposing}$$

Exercise 7.12  $L_1 = 1.6H$ ,  $L_2 = 4.9H$

a)  $L = L_1 + L_2 + k\sqrt{L_1 L_2}$

$= 6.5 + 2.8k$  (series-aiding)

b) (Series-opposing)

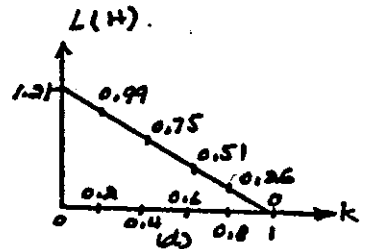
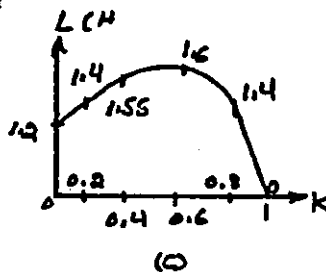
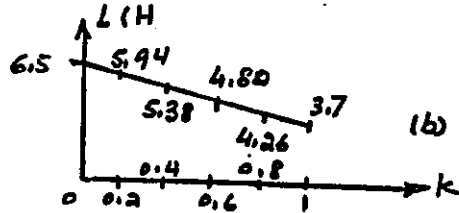
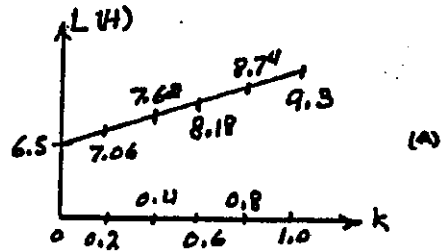
$L = 6.5 - 2.8k$

c) Parallel-aiding

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{7.84 - 7.84k^2}{6.5 - 5.6k}$$

d) Parallel-opposing

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{7.84 - 7.84k^2}{6.5 + 5.6k}$$



Exercise 7.13  $\vec{J} = \int_0^a \vec{a}_2 A/m^2$   $P \leq a \Rightarrow \oint \vec{H} \cdot d\vec{l} = I_{enc}$  or  
 $= 0$   $P > a$   $H_\phi = \frac{J_0 P}{s}$   $P \leq a$  (Inside)

$$\omega_m = \frac{1}{2} \mu_0 H_\phi^2 = \frac{1}{8} \mu_0 J_0^2 P^2 \Rightarrow W_m = \frac{1}{8} \mu_0 J_0^2 \int_0^a P^2 dP \int_0^{2\pi} d\phi \int_0^L dz = \frac{\mu_0 J_0^2 a^4}{16} J/m$$

Outside;  $P > a$   $H_\phi = \frac{J_0 a^2}{2P} \Rightarrow \omega_m = \frac{1}{8} \mu_0 \left(\frac{J_0}{P}\right)^2 a^4 J/m^3$

Exercise 7.14  $\Phi = a\sqrt{i}$ ,  $d\Phi = \frac{1}{2} a \frac{di}{\sqrt{i}}$ ,  $dW = N i d\Phi = \frac{1}{2} a N \sqrt{i} di$

$$W = \int_0^i \frac{1}{2} a N \sqrt{i} di = \frac{1}{3} a N i^{3/2} = \frac{1}{3} N \Phi i \text{ (Non-linear medium)}$$

Exercise 7.15

$$W_i = \frac{1}{2} \times 1.02 \times 2^2 = 2.04 J$$

$$W_f = \frac{1}{2} \times 1.02 \times 5^2 = 12.75 J$$

Thus,  $W = W_f - W_i = 10.71 J$

Exercise 7.16  $\vec{J} = J_0 \cos \omega t \vec{a}_z \Rightarrow \vec{E} = \frac{\vec{J}}{\sigma}$ ,  $\vec{D} = \epsilon \vec{E} \Rightarrow \vec{D} = \frac{\epsilon}{\sigma} J_0 \cos \omega t \vec{a}_z$

$$\frac{\partial \vec{D}}{\partial t} = -\frac{\omega \epsilon}{\sigma} J_0 \sin \omega t \vec{a}_z \Rightarrow \frac{|\frac{\partial \vec{D}}{\partial t}|}{|\vec{J}|} = \frac{\omega \epsilon}{\sigma} \quad \omega = 2\pi f$$

For Cu:  $\epsilon \rightarrow \epsilon_0 = 10^{-9}/36\pi \text{ F/m}$ ,  $\sigma = 5.8 \times 10^7 \text{ S/m} \Rightarrow \frac{\omega \epsilon}{\sigma} = 9.58 \times 10^{-9} f$

Exercise 7.17  $\vec{E} = C \cos(\alpha x) \cos(\omega t - \beta z) \vec{a}_y$ , since  $\vec{D} = \epsilon \vec{E}$

$$\vec{D} = \epsilon C \cos(\alpha x) \cos(\omega t - \beta z) \vec{a}_y \quad \text{and} \quad \nabla \cdot \vec{D} = 0$$

From  $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ , we have

$$-\frac{\partial \vec{B}}{\partial t} = \beta C \cos(\alpha x) \sin(\omega t - \beta z) \vec{a}_x - \alpha C \sin(\alpha x) \cos(\omega t - \beta z) \vec{a}_z$$

$$\text{or } \vec{B} = -\frac{\beta C}{\omega} \cos(\alpha x) \cos(\omega t - \beta z) \vec{a}_x + \frac{\alpha C}{\omega} \sin(\alpha x) \sin(\omega t - \beta z) \vec{a}_z \quad \tau$$

Thus,  $\vec{H} = -\frac{\beta C}{\omega \mu} \cos(\alpha x) \sin(\omega t - \beta z) \vec{a}_x + \frac{\alpha C}{\omega \mu} \sin(\alpha x) \sin(\omega t - \beta z) \vec{a}_z \quad \text{A/m}$

And  $\nabla \cdot \vec{B} = 0$

Exercise 7.18

$$\oint_S \vec{B} \cdot d\vec{s} = 0 \Rightarrow \Delta \vec{s} = \vec{a}_n \Delta s$$

$$\lim_{h \rightarrow 0} [\vec{B}_1 \cdot \Delta \vec{s} - \vec{B}_2 \cdot \Delta \vec{s}] = 0$$

$$\vec{a}_n \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

Similarly,  $\oint_S \vec{D} \cdot d\vec{s} = \int_V \rho_v dV$

$$\lim_{h \rightarrow 0} [\vec{D}_1 \cdot \Delta \vec{s} - \vec{D}_2 \cdot \Delta \vec{s}] = \rho_s \Delta s$$

$$\vec{a}_n \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

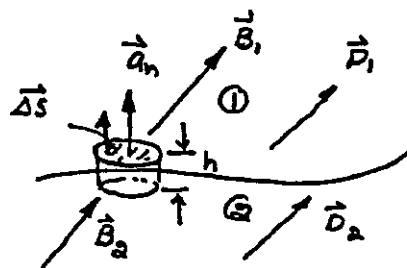
$$\oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\text{as } h \rightarrow 0 \quad ds \rightarrow 0 \Rightarrow \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \rightarrow 0$$

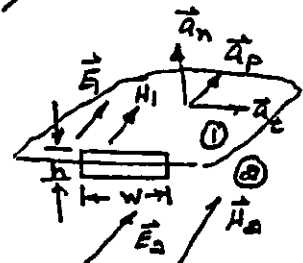
Thus  $\vec{E}_1 \cdot \vec{a}_t w - \vec{E}_2 \cdot \vec{a}_t w = 0$

$$\text{or } \vec{a}_t \cdot (\vec{E}_1 - \vec{E}_2) = 0 \Rightarrow$$

$$\vec{a}_n \times (\vec{E}_1 - \vec{E}_2) = 0$$



$$\lim_{h \rightarrow 0} \rho_v = \rho_s$$



$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

$$\text{as } h \rightarrow 0 \quad \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s} = 0$$

$$\text{and } \lim_{h \rightarrow 0} \vec{J} = \vec{J}_s$$

Thus  $\vec{a}_t \cdot (\vec{H}_1 - \vec{H}_2) = \vec{J}_s \quad \text{or}$

$$\vec{a}_n \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

Exercise 7.19 Medium-2 is a perfect conductor  $\Rightarrow \vec{E}_2 = 0, \vec{B}_2 = 0, \vec{D}_2 = 0, \vec{H}_2 = 0$

(a) Thus,  $D_{n1} = \rho_s \quad B_{n1} = 0 \quad E_{t1} = 0 \quad H_{t1} = J_s$

(b) Two perfect dielectrics:  $\rho_s = 0 \quad J_s = 0$

$$D_{n1} = D_{n2}, \quad B_{n1} = B_{n2} \quad E_{t1} = E_{t2} \quad H_{t1} = H_{t2}$$

(c) Medium 2 is a conductor,  $J_s = 0$

$$\vec{a}_n \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s \quad \vec{a}_n \cdot (\vec{B}_1 - \vec{B}_2) = 0 \quad \vec{a}_n \times (\vec{E}_1 - \vec{E}_2) = 0, \quad \vec{a}_n \times (\vec{H}_1 - \vec{H}_2) = 0$$

Exercise 7.20  $\vec{E} = C \cos(\alpha x) \cos(\omega t - \beta z) \vec{a}_y$ , from Exercise 7.17,

$$\vec{H} = -\frac{\beta C}{\omega \mu} \cos(\alpha x) \cos(\omega t - \beta z) \vec{a}_x + \frac{\alpha C}{\omega \mu} \sin(\alpha x) \sin(\omega t - \beta z) \vec{a}_z$$

$$\text{However, } \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \Rightarrow \vec{a}_y \left[ \frac{\partial}{\partial z} H_x - \frac{\partial}{\partial x} H_z \right] = \frac{\partial}{\partial t} (\epsilon \vec{E})$$

$$-\frac{\beta^2 C}{\omega \mu} \cos(\alpha x) \sin(\omega t - \beta z) - \frac{\alpha^2 C}{\omega \mu} \cos(\alpha x) \sin(\omega t - \beta z) = -\omega \epsilon C \cos(\alpha x) \sin(\omega t - \beta z)$$

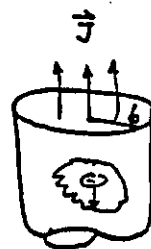
$$\text{or } \beta^2 + \alpha^2 = \omega^2 \mu \epsilon \quad \text{condition for fields to exist.}$$

Exercise 7.21  $\vec{J} = \frac{I}{\pi b^2} \vec{a}_z \quad \text{A/m}^2$

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{I}{\sigma \pi b^2} \vec{a}_z \quad \text{V/m}$$

$$P = \int_V \vec{J} \cdot \vec{E} \, dv = \frac{I^2}{\sigma \pi^2 b^4} \int_0^b \rho \, d\rho \int_0^{2\pi} d\phi \int_0^L dz$$

$$= I^2 \frac{L}{\sigma \pi b^2} = I^2 R \quad \text{where } R = \frac{L}{\sigma \pi b^2} \, \Omega$$



On the surface of the conductor:  $\vec{H} = \frac{I}{2\pi b} \vec{a}_\phi$

$$\text{Thus, } \vec{S} = \vec{E} \times \vec{H} = -\frac{I^2}{2\sigma \pi^2 b^3} \vec{a}_\rho \quad \text{W/m}^2$$

$$\int_S \vec{S} \cdot d\vec{s} = -\frac{I^2}{2\sigma \pi^2 b^3} \int_0^{2\pi} b \, d\phi \int_0^L dz = -I^2 \frac{L}{\sigma \pi b^2} = -I^2 R$$

The minus sign indicates that the power is flowing into the volume  $V$  of the solid conductor.

Exercise 7.22  $\vec{E} = 10 \cos(\omega t + ky) \vec{a}_x$  V/m  $T = 100 \text{ ns} \Rightarrow f = 10 \text{ MHz}$   
 $\vec{D} = 10 \epsilon_0 \cos(\omega t + ky) \vec{a}_x$  C/m<sup>2</sup>,  $\nabla \cdot \vec{D} = 0$   $\omega = 2\pi f$

$$-\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E} = -\frac{\partial E_x}{\partial y} \vec{a}_z = 10k \sin(\omega t + ky) \vec{a}_z \Rightarrow \vec{B} = + \frac{10k}{\omega} \cos(\omega t + ky) \vec{a}_z$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{H} \Rightarrow -10\omega\epsilon_0 \sin(\omega t + ky) \vec{a}_x = -\frac{10k^2}{\omega\mu_0} \sin(\omega t + ky) \vec{a}_x \quad \nabla \cdot \vec{B} = 0$$

$$\text{or } k^2 = \omega^2 \mu_0 \epsilon_0 \Rightarrow k = \pm \omega \sqrt{\mu_0 \epsilon_0} = \pm 2\pi \times 10 \times 10^6 / 3 \times 10^8 = \pm 0.209 \text{ rad/m}$$

Exercise 7.23  $\vec{E}_x = E e^{-jkz} \Rightarrow \vec{D}_x = \epsilon E e^{-jkz}$  and  $\nabla \cdot \vec{D} = 0$

$$\nabla \times \vec{E} = \frac{\partial \vec{E}_x}{\partial z} \vec{a}_y = -jkE e^{-jkz} \vec{a}_y$$

$$\text{Thus, from } \nabla \times \vec{E} = -j\omega \vec{B} \Rightarrow \vec{B} = \frac{kE}{\omega} e^{-jkz} \vec{a}_y \text{ and } \nabla \cdot \vec{B} = 0$$

$$\vec{H}_y = \frac{kE}{\omega\mu} e^{-jkz} \Rightarrow H_y = \frac{kE}{\omega\mu} \cos(\omega t - kz)$$

$$\nabla \times \vec{H} = -\frac{\partial (\vec{H}_y)}{\partial x} \vec{a}_z = j \frac{k^2}{\omega\mu} e^{-jkz}$$

$$\text{From } \nabla \times \vec{H} = j\omega \vec{D}, \text{ we have } k^2 = \omega^2 \mu \epsilon$$

$$\text{Finally, } \langle \hat{S} \rangle = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] = \frac{1}{2} \frac{k}{\omega\mu} E^2 \vec{a}_z$$

Exercise 7.24  $\vec{E}_x = 10 e^{jky}$ ,  $\vec{D}_x = 10 \epsilon_0 e^{jky} \Rightarrow \nabla \cdot \vec{D} = 0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{E}_x}{\partial y} \vec{a}_z = -j10k e^{jky} \vec{a}_z, \text{ Since } \nabla \times \vec{E} = -j\omega \vec{B}, \Rightarrow \vec{B}_z = \frac{10k}{\omega} e^{jky} \text{ and } \nabla \cdot \vec{B} = 0$$

$$\vec{H}_z = \frac{10k}{\omega\mu_0} e^{jky} \quad \nabla \times \vec{H} = \frac{\partial \vec{H}_z}{\partial y} \vec{a}_x = j \frac{10}{\omega\mu_0} k^2 e^{jky} \vec{a}_x \quad c = 3 \times 10^8 \text{ m/s}$$

$$\text{From } \nabla \times \vec{H} = j\omega \vec{D}, \text{ we have } k^2 = \omega^2 \mu_0 \epsilon_0 \text{ or } k = \pm \omega \sqrt{\mu_0 \epsilon_0} = \pm \frac{\omega}{c}$$

$$T = 100 \text{ ns}, f = 10 \text{ MHz and } k = \pm 0.209 \text{ rad/m.}$$

$$\langle \hat{S} \rangle = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] = -\frac{50k}{\omega\mu_0} \vec{a}_y = -0.133 \vec{a}_y \text{ W/m}^2$$

### Exercise 7.25

$$S = 10 \text{ kVA}, V_1 = 500 \text{ V} \quad a = a \Rightarrow V_2 = 500/a = 250 \text{ V}$$

$$P_0 = 8000 \times 0.8 = 6400 \text{ W}$$

$$I_2 = \frac{8000}{250} = 32 \text{ A}, I_1 = \frac{I_2}{a} = 16$$

$$P_f = 0.8 (\text{lead}) \Rightarrow \theta = +36.87^\circ \quad \vec{I}_2 = 32 \angle 36.87^\circ \text{ A}, \vec{I}_1 = 16 \angle 36.87^\circ \text{ A}$$



### Exercise 7.26

$$L_2 = \frac{N_2^2}{R} \quad L_1 = \frac{N_1^2}{R} \quad L_1 = 80 \text{ mH}$$

$$L_2 = \left(\frac{N_2}{N_1}\right)^2 L_1 = 20 \text{ mH}$$

$$k=1 \Rightarrow M = \sqrt{L_1 L_2} = 40 \text{ mH}$$

$$\hat{Z}_1 = j\omega(L_1 - M) = j15.08\Omega, \quad \hat{Z}_2 = j\omega(L_2 - M) = -j7.54\Omega, \quad \hat{Z}_M = j\omega M = j15.08\Omega$$

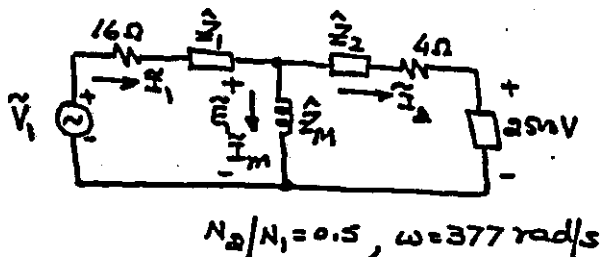
$$\tilde{V}_2 = 2500 \angle 0^\circ \text{ V}, \quad \tilde{I}_2 = 40 \angle -45^\circ \text{ A}$$

$$\tilde{E}_1 = 2500 + 40 \angle -45^\circ (4 - j7.54) = 2421.98 \angle -7.74^\circ \text{ V}$$

$$\tilde{I}_M = \frac{\tilde{E}_1}{\hat{Z}_M} = 160.61 \angle -97.74^\circ \text{ A}, \quad \tilde{I}_1 = \tilde{I}_M + \tilde{I}_2 = 187.54 \angle -87.94^\circ \text{ A}$$

$$\tilde{V}_1 = \tilde{E}_1 + \tilde{I}_1 (16 + \hat{Z}_1) = 6231.84 \angle -31.17^\circ \text{ V}$$

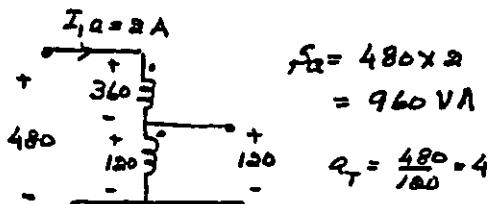
$$P_{in} = R_1 [\tilde{V}_1 \tilde{I}_1^*] = 640 \text{ kW} \quad P_o = R_2 [\tilde{V}_2 \tilde{I}_2^*] = 70.7 \text{ kW}, \quad \eta = \frac{P_o}{P_{in}} = 0.11 \text{ or } 11\%$$



### Exercise 7.27

$$I_1 = 720/360 = 2 \text{ A}$$

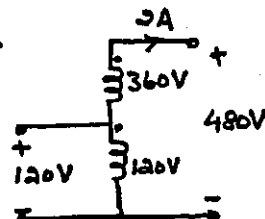
$$I_2 = 720/120 = 6 \text{ A}$$



$$S_2 = 480 \times 2 = 960 \text{ VA}$$

$$Q_T = \frac{480}{120} = 4$$

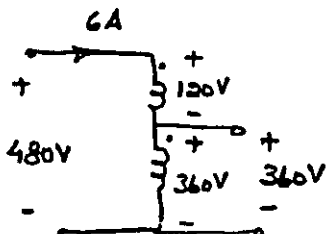
$$480/120 \text{ V}, 960 \text{ VA}$$



$$Q_T = 120/480 = 0.25$$

$$S_2 = 480 \times 2 = 960 \text{ VA}$$

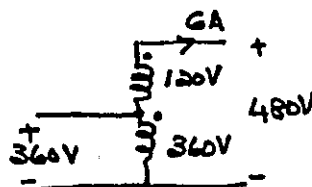
$$120/480 \text{ V}, 960 \text{ VA}$$



$$Q_T = 480/360 = 1.333$$

$$S_2 = 480 \times 6 = 2880 \text{ VA}$$

$$480/360 \text{ V}, 2880 \text{ VA}$$



$$Q_T = 360/480 = 0.75$$

$$S_2 = 480 \times 6 = 2880 \text{ VA}$$

$$360/480 \text{ V}, 2880 \text{ VA}$$

# Exercise 7.28

From Exercise 7.27,

$$a_T = 4, \quad \tilde{I}_{1a} = 2 \angle -36.87^\circ \text{ A}$$

$$\tilde{I}_{2a} = 4\tilde{I}_{1a} = 8 \angle -36.87^\circ \text{ A}$$

$$\tilde{I}_2 = 6 \angle -36.87^\circ \text{ A}$$

$$P_0 = 120 \times 8 \times 0.8 = 768 \text{ W}$$

$$P_{in} = P_0 + I_{1a}^2 (4.5) + I_2^2 (0.5) = 804 \text{ W}$$

$$\eta = \frac{768}{804} = 0.9552 \text{ or } 95.52\%$$

$$\tilde{I}_{1a} = 6 \angle -36.87^\circ \text{ A}, \quad \tilde{I}_1 = 2 \angle -36.87^\circ \text{ A}$$

$$\tilde{I}_{2a} = 8 \angle -36.87^\circ \text{ A}$$

$$P_0 = 360 \times 8 \times 0.8 = 2304 \text{ W}$$

$$P_{in} = P_0 + I_{1a}^2 (0.5) + I_1^2 (4.5) = 2340 \text{ W}$$

$$\eta = \frac{P_0}{P_{in}} = 0.9846 \text{ or } 98.46\%$$

$$\tilde{I}_{2a} = 6 \angle -36.87^\circ \text{ A}$$

$$\tilde{I}_1 = 2 \angle -36.87^\circ \text{ V}$$

$$P_0 = 480 \times 6 \times 0.8 = 2304 \text{ W}$$

$$P_{in} = 2304 + 2^2 \times 4.5 + 6^2 \times 0.5 = 2340 \text{ W} \Rightarrow \eta = 98.46\%$$

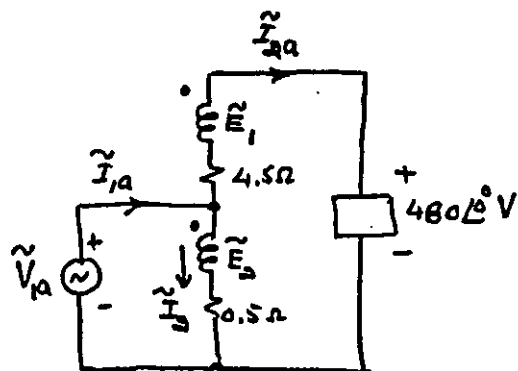
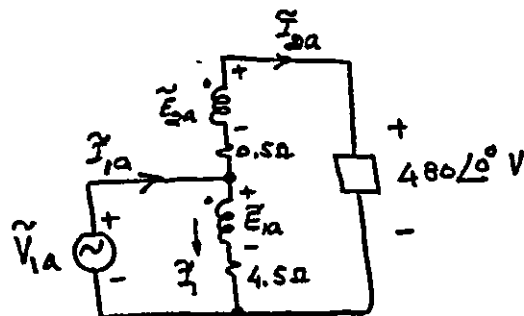
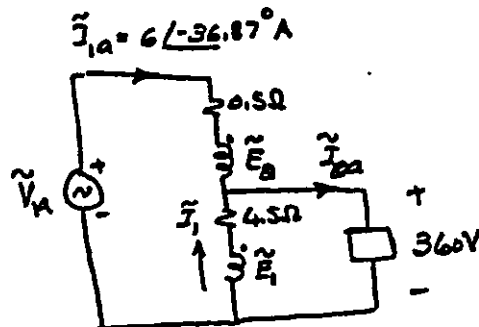
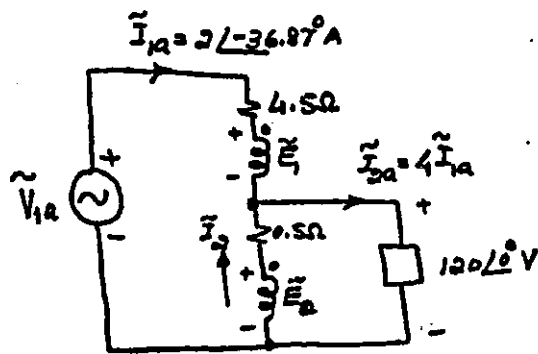
$$\tilde{I}_{2a} = 2 \angle -36.87^\circ \text{ A}$$

$$\tilde{I}_{1a} = 8 \angle -36.87^\circ \text{ A}, \quad \tilde{I}_2 = 6 \angle -36.87^\circ \text{ A}$$

$$P_0 = 480 \times 2 \times 0.8 = 768 \text{ W}$$

$$P_{in} = 768 + 6^2 \times 0.5 + 2^2 \times 4.5 = 804 \text{ W}$$

$$\eta = 768/804 = 0.9552 \text{ or } 95.52\%$$



Exercise 7.29at  $r=a$ ,  $\vec{B} = B_m \sin \omega t \vec{a}_z$ 

$$\text{From (7.153)} \quad \vec{B}_0 = \oint \vec{B} = \oint B_m \sin \omega t \vec{a}_z$$

$$\text{Thus,} \quad \Phi = \oint B_m \pi a^2 \sin \omega t$$

$$e = - \frac{d\Phi}{dt} = - \oint \pi a^2 \omega B_m \cos \omega t$$

$$\text{Hence, } \vec{E} = \frac{e}{\oint \pi a} = - a \omega B_m \cos \omega t \vec{a}_\phi$$

$$\text{Force: } \vec{F} = -e\vec{E} = a e \omega B_m \cos \omega t \vec{a}_\phi$$

$$\text{Work done per revolution: } W = \oint a \vec{a} \cdot \vec{F} = \oint \pi a^2 e \omega \cos \omega t$$

Exercise 7.30 $B_m = 0.4 \text{ T} \quad a = 0.84 \text{ m} \quad f = 60 \text{ Hz}, \omega \approx 377 \text{ rad/s}$ 

$$B_0 = 0.8 \sin \omega t \text{ (T)} \quad \Phi = 1.773 \sin \omega t \text{ (Wb)}$$

$$E_\phi = -126.67 \cos \omega t \text{ (V/m)}, \quad W = 1.07 \times 10^{16} \cos \omega t \text{ J}$$

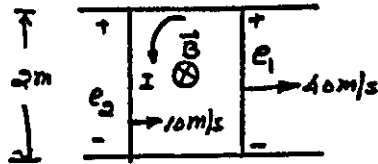
$$W_{\text{avg}} = \frac{2}{\pi} (1.07 \times 10^{16}) = 6.81 \times 10^{17} \text{ J or } 425.6 \text{ eV}$$

### Problem 7.1

$$e_1 = 0.8 \times 2 \times 40 = 64 \text{ V}$$

$$e_2 = 0.8 \times 2 \times 10 = 16 \text{ V}$$

$$I = \frac{64-16}{12} = 4 \text{ A}$$



### Problem 7.2

$$0.5 \text{ s} \leq 1.25 \text{ s}$$

$$u_y = 100 \text{ m/s} \Rightarrow 0 \leq t \leq 12.5 \text{ ms}$$

Flux linking the loop is constant.

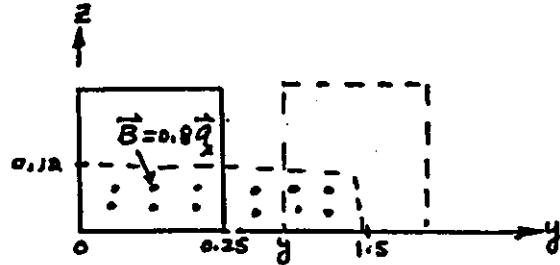
$$\Phi = 0.8 \times 0.12 \times 0.25 = 24 \text{ mWb}$$

$$e = - \frac{d\Phi}{dt} = 0 \Rightarrow I = 0$$

$$1.25 \leq y \leq 1.5 \text{ m} \quad \text{or} \quad 12.5 \leq t \leq 15 \text{ ms}$$

$$\Phi = 0.8 \times 0.12 \int_{t=0}^{15 \text{ ms}} 100 \text{ m/s} dt = 0.144 - 9.6t \Rightarrow e = - \frac{d\Phi}{dt} = 9.6 \text{ V}, I = \frac{9.6}{12} = 0.8 \text{ A}$$

$$y \geq 1.5 \text{ m} \text{ or } t \geq 15 \text{ ms}, \Phi = 0 \Rightarrow e = 0 \text{ and } I = 0$$



### Problem 7.3

$$L = 0.2 \text{ m}, \omega = \frac{2\pi \times 1000}{60} = 40\pi \quad B_z = 250 \cos 30^\circ = 216.5 \text{ mT}$$

Hence,  $e = \frac{1}{2} B_z \omega L^2 = 0.54 \text{ V}$

### Problem 7.4

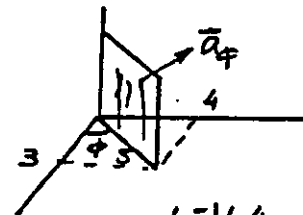
$$\eta = 50,000 \text{ Turns/unit length} \quad I = 12 \text{ A}, L = 3 \text{ cm}$$

$$\omega = \frac{2\pi \times 3600}{60} = 120\pi \text{ rad/s} \quad e = \frac{1}{2} B \omega L^2 = \frac{1}{2} (\mu_0 \eta I) \omega L^2 \approx 0.13 \text{ V}$$

### Problem 7.5

$$\vec{a}_\phi = -\vec{a}_x \sin 53.13^\circ + \vec{a}_y \cos 53.13^\circ$$

$$= -0.8 \vec{a}_x + 0.6 \vec{a}_y$$



$$\theta = \tan^{-1}(\frac{4}{3}) = 53.13^\circ$$

$$d\vec{s} = d\rho dz \vec{a}_\phi$$

$$\vec{B} \cdot d\vec{s} = -2 \sin 300t + 1.05 \cos 300t \text{ (mWb)}$$

$$\Phi = \int \vec{B} \cdot d\vec{s} = \int_0^5 d\rho \int_0^4 (-2 \sin 300t + 1.05 \cos 300t) dz$$

$$= -40 \sin 300t + 21 \cos 300t \text{ mWb}$$

$$e = - \frac{d\Phi}{dt} = 12 \cos 300t + 6.3 \sin 300t \text{ (V)}$$

$$i = \frac{e}{5} = 6 \cos 300t + 3.15 \sin 300t$$

$$= 6.78 \cos(300t - 27.67^\circ) \text{ A}$$

Problem 7.6  $\vec{B} = 0.8 \vec{a}_y$  T

$$\Phi = \int_S \vec{B} \cdot d\vec{s} = 0.8 A \cos \theta$$

$$= 3.2 \cos \omega t \text{ mWb}$$

$$e = -N \frac{d\Phi}{dt} = 3.2 N \omega \sin \omega t \text{ (mV)}$$

$$= 76.8 \sin 120t, \text{ V}$$

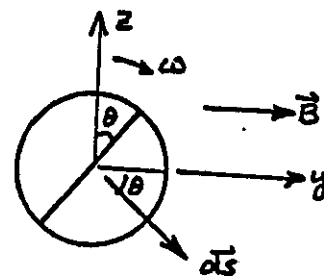
$$\theta = \omega t$$

Area:

$$A = 40 \text{ cm}^2$$

$$N = 200$$

$$\omega = 120 \text{ rad/s}$$



Problem 7.7  $\vec{B} = 0.8 \sin 120t$  T  $\Phi = 0.8 \sin(120t) \cos(120t) \int d\vec{s}$

$$= 1.6 \sin 240t \text{ mWb}$$

$$e = -N \frac{d\Phi}{dt} = -200(1.6 \times 10^{-3}) 240 \cos 240t$$

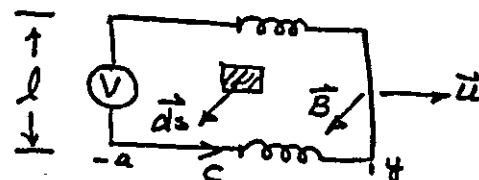
$$= -76.8 \cos 240t = 76.8 \sin(240t - 90^\circ) \text{ V}$$

Problem 7.8  $\vec{u} = u \cos \omega t \vec{a}_y$  m/s

$$\vec{B} = B \cos \omega t \vec{a}_x$$
 T

$$e_m = \int_C (\vec{u} \times \vec{B}) \cdot d\vec{l} = -Bu \cos^2 \omega t \int_0^l dz$$

$$= -Bul \cos^2 \omega t$$



$$e_r = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} = B \omega \sin \omega t \int_0^l dz \int_{-a}^y dy = B \omega l (y+a) \sin \omega t$$

But  $\frac{dy}{dt} = u \cos \omega t \Rightarrow y = \frac{u}{\omega} \sin \omega t + \text{const}$  ( $\because t=0, y=0$ )

Thus,  $e = e_m + e_r = -Bul \cos^2 \omega t + Bul \sin^2 \omega t + B \omega l a \sin \omega t$

$$= B \omega l a \sin \omega t - Bul \cos 2\omega t \text{ (V)}$$

b) Faraday's law:  $d\Phi = \vec{B} \cdot d\vec{s} = B \cos \omega t dy dz$

$$\Phi = B \cos \omega t \int_{-a}^y dy \int_0^l dz = B l y \cos \omega t + B l a \cos \omega t$$

$$e = - \frac{d\Phi}{dt} = B l y \omega \sin \omega t - B l \cos \omega t \frac{dy}{dt} + B l a \omega \sin \omega t$$

Since  $\frac{dy}{dt} = u \cos \omega t$  and  $y = \frac{u}{\omega} \sin \omega t$

$$e = B l a \omega \sin \omega t + B l u \sin^2 \omega t - B l u \cos^2 \omega t$$

$$= B l a \omega \sin \omega t - B l u \cos 2\omega t, \text{ V}$$

Problem 7.9  $\vec{u} = -1000 \vec{a}_y \text{ m/s}$   $\vec{B} = 0.2 \vec{a}_z \text{ T}$

$$\vec{u} \times \vec{B} = -200 \vec{a}_x \Rightarrow \mathcal{E} = \int_{0.1}^0 (\vec{u} \times \vec{B}) \cdot d\vec{l} = 20 \text{ V}$$

Problem 7.10  $\mathcal{F} = NI = 1200 \times 0.75 = 900 \text{ At}$

From Figure 5.37,  $B = 1.05 \text{ T}$ ,  $\mathcal{F} = BA = 1.05 \times 10^{-4} \times 4 = 4.2 \times 10^{-4} \text{ Wb}$

$$L = \frac{N\mathcal{F}}{I} = \frac{1200 \times 4.2 \times 10^{-4}}{0.75} = 0.672 \text{ H or } 672 \text{ mH}$$

Problem 7.11  $\mathcal{F} = \frac{LI}{N} = 20 \times 10^3 \times 2.5 / 1000 = 50 \text{ mWb}$

Problem 7.12 (a)  $L = \frac{\mu_0}{2\pi} \ln(b/a) = \frac{4\pi \times 10^{-7}}{2\pi} \ln(4/2) = 138.63 \text{ nH}$

(b)  $L = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \ln(b/a) = 50 + 138.63 = 188.63 \text{ nH}$

Problem 7.13  $R = \frac{l}{\mu A} = \frac{80 \times 10^{-2}}{4\pi \times 10^{-7} \times 500 \times 40 \times 10^{-4}} = 318,309.89 \text{ } (\Omega)$

$$L_{11} = \frac{N_1^2}{R} = \frac{100^2}{R} = 31.42 \text{ mH}, L_{22} = \frac{N_2^2}{R} = \frac{150^2}{R} = 70.69 \text{ mH}, L_{33} = \frac{N_3^2}{R} = \frac{200^2}{R} = 126 \text{ mH}$$

$$L_{12} = L_{21} = \frac{N_1 N_2}{R} = 47.12 \text{ mH}, L_{13} = L_{31} = \frac{N_1 N_3}{R} = 62.83 \text{ mH}, L_{23} = L_{32} = \frac{N_2 N_3}{R} = 94.25 \text{ mH}$$

$$i_1 = 10 \sin(800\pi t) \Rightarrow \frac{di_1}{dt} = 25132.74 \cos(800\pi t)$$

$$e_1 = L_{11} \frac{di_1}{dt} = 789.67 \cos(800\pi t) \text{ V}, e_2 = L_{12} \frac{di_1}{dt} = 1184.3 \cos(800\pi t) \text{ V}$$

$$e_3 = L_{13} \frac{di_1}{dt} = 1579.1 \cos(800\pi t) \text{ V}$$

Problem 7.14

$$\left. \begin{aligned} L_1 + L_2 + 2M &= 3.28 \text{ (mH)} \\ L_1 + L_2 - 2M &= 0.72 \text{ (mH)} \end{aligned} \right\} = M = 0.64 \text{ mH}$$

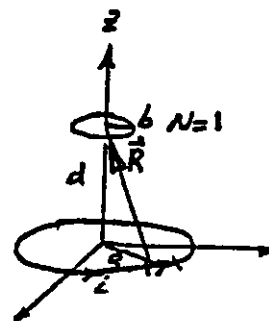
Let  $L_1 = 4L_2$ , then  $5L_2 + 2M = 3.28 \Rightarrow L_2 = 0.4 \text{ mH}$

$$k = \frac{M}{\sqrt{L_1 L_2}} = 0.8$$

$$L_1 = 1.6 \text{ mH}$$

### Problem 7.15

$$\begin{aligned}\vec{B} &= \frac{\mu_0 i}{4\pi} \int_0^{2\pi} \frac{a d\phi (\vec{a}_\phi \times \vec{a}_r)}{R^2} \\ &= \frac{\mu_0 i}{4\pi} \int_0^{2\pi} \frac{a^2 d\phi}{R^3} \vec{a}_z + \frac{\mu_0 i}{4\pi} a \int_0^{2\pi} \frac{d\phi}{R^3} \vec{a}_\phi \\ &= \frac{\mu_0 i}{2} \frac{a^2}{R^3} \quad R = \sqrt{a^2 + d^2}\end{aligned}$$



$$\vec{R} = -a \vec{a}_\phi + d \vec{a}_z$$

$$\Phi = \int_S \vec{B} \cdot d\vec{s} = B_z \pi b^2 = \frac{\mu_0 i}{2} \frac{\pi b^2 a^2}{(a^2 + d^2)^{3/2}}$$

$$L_{12} = L_{21} = M = \frac{N\Phi}{I} = \frac{\mu_0}{2} \frac{\pi a^2 b^2}{(a^2 + d^2)^{3/2}}$$

Problem 7.16 Same as Problem 7.15 except  $d=0$ . Thus,

$$M = \frac{\mu_0}{2a} \pi b^2$$

Problem 7.17  $B_\phi = \frac{\mu_0 i}{2\pi r}$  but  $P = r \sin\theta$   $d\vec{s} = r dr d\theta \vec{a}_\phi$

$$\begin{aligned}\Phi &= \frac{\mu_0 i}{2\pi} \int_a^b dr \int_{\pi/6}^{\pi/2} \frac{d\theta}{\sin\theta} = \frac{\mu_0 i}{2\pi} (b-a) \left[ \ln \left( \tan \frac{\theta}{2} \right) \right]_{\pi/6}^{\pi/2} \\ &= 0.21 \mu_0 (b-a) i\end{aligned}$$

Thus,  $M = 0.21 \mu_0 (b-a)$

Problem 7.18  $\Phi = BA = \mu_0 N_1 i_1 A \Rightarrow M = \frac{N_2 \Phi}{i_2} = N_1 N_2 \mu_0 A$

$$= 4\pi \times 10^{-7} \times 400 \times 4000 \times \pi \times 10^{-4} = 631.65 \mu\text{H/m}$$

$$e \Rightarrow M \frac{di_1}{dt} = -631.65 \times 10^{-6} \times 0.5 \times 200 \sin 200t = -63.16 \sin 200t, \text{ mV/m}$$

Problem 7.19

$$\begin{aligned}\Phi &= a i^n \quad d\Phi = a n i^{n-1} di \\ dW &= N i d\Phi = N a n i^n di \quad \left[ \begin{array}{l} p = \partial i = N \frac{d\Phi}{dt} \\ dW = p dt = N i d\Phi \end{array} \right] \\ W &= N a n \int_0^I i^n di = N a \left( \frac{n}{n+1} \right) I^{n+1} = N \left( \frac{n}{n+1} \right) \Phi I\end{aligned}$$

Problem 7.20  $\Phi = a \ln(bi) \Rightarrow d\Phi = \frac{a}{i} di$

$dW = Ni d\Phi = Na di \Rightarrow W = Na \int_0^I di = NaI$

Problem 7.21  $\Phi = \frac{ai}{b+ci} \Rightarrow d\Phi = \frac{ba di}{(b+ci)^2}$

$W = Nba \int_0^I \frac{i}{(b+ci)^2} di = \frac{Nba}{c^2} \left[ \ln(b+ci) + \frac{b}{b+ci} \right]_0^I$   
 $= \frac{Nba}{c^2} \left[ \ln\left(\frac{b+cI}{b}\right) - \frac{cI}{b+cI} \right]$

Problem 7.22  $I = 200/10 = 20 \text{ A}$   $W_m = \frac{1}{2} LI^2 = \frac{1}{2} \times 30 \times 10^{-3} \times 20^2 = 6 \text{ J}$

$w_m = \frac{6}{20 \times 10^{-2} \times \pi \times 2.5^2 \times 10^{-4}} = 15.28 \text{ kJ/m}^3$

Since  $w_m = \frac{1}{2} \frac{B^2}{\mu_0} \Rightarrow B = \sqrt{2 \times 4\pi \times 10^{-7} \times 15.28 \times 10^3} \approx 0.196 \text{ T}$

Problem 7.23  $H\phi = \frac{I}{2\pi\rho} \Rightarrow w_m = \frac{\mu_0 I^2}{8\pi^2 \rho^2} = \frac{4\pi \times 10^{-7} \times 1000^2}{8\pi^2 \rho^2} = \frac{0.0159}{\rho^2} \text{ J/m}^3$

$W = 0.0159 \int_{0.05}^{0.1} \frac{1}{\rho} d\rho \int_0^{2\pi} d\phi = 64.3 \text{ mJ/m}$  [Energy per unit length]

Problem 7.24  $B = 0.04 \text{ mT} \Rightarrow w_m = \frac{1}{2} \frac{(0.04 \times 10^{-3})^2}{4\pi \times 10^{-7}} = 636.62 \text{ } \mu\text{J/m}^3$

$W = 636.62 \times 10^{-6} \times \frac{4\pi}{3} [12.8^3 - 6.4^3] 10^{12} = 4.89 \times 10^{12} \text{ J}$

Problem 7.25  $R = 0.5 \Omega$ ,  $L = 2 \text{ H}$   $W = 6.4 \text{ kJ}$

$W = \frac{1}{2} LI^2 \Rightarrow I = \sqrt{\frac{2 \times 6400}{2}} = 80 \text{ A}$

Thus,  $P = I^2 R = 3.2 \text{ kW}$



Problem 7.26  $N=500$  Turns  $a=10$  cm,  $b=15$  cm,  $\mu_r=1000$   $I=10$  A

$$\mu_0 = \frac{\mu_r}{2\pi} \Rightarrow \omega_m = \frac{\mu N^2 I^2}{8\pi^2 r^2} = \frac{397.89}{r^2} \text{ J/m}^3$$

$$W_m = 397.89 \int_{0.1}^{0.15} \frac{1}{r} dr \int_0^{2\pi} d\theta \int_0^{0.05} dz = 397.89 (2\pi) (0.05) \ln(1.5) = 50.68 \text{ J}$$

$$\text{Since } W_m = \frac{1}{2} LI^2 \Rightarrow L = \frac{2 \times 50.68}{10^2} = 1.014 \text{ H}$$

Problem 7.27  $\nabla \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} = 0$   $\vec{J} = \sigma \vec{E} = \frac{\sigma}{\epsilon} \vec{D}$   $\nabla \cdot \vec{J} = \frac{\sigma}{\epsilon} \nabla \cdot \vec{D} = \frac{\sigma}{\epsilon} \rho_v$

$$\text{Hence } \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon} \rho_v = 0$$

Problem 7.28

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad \text{Since } \vec{B} = \nabla \times \vec{A},$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t} (\nabla \times \vec{A}) = 0 \Rightarrow \nabla \times \vec{E} + \nabla \times \left( \frac{\partial \vec{A}}{\partial t} \right) = 0$$

$$\text{or } \nabla \times \left( \vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0 \Rightarrow \oint_c \left( \vec{E} + \frac{\partial \vec{A}}{\partial t} \right) \cdot d\vec{l} = 0 \quad [\text{Stokes' Thm}]$$

Problem 7.29  $\vec{E} = [E_0 \sin(\alpha x - \omega t) + E_0 \sin(\alpha x + \omega t)] \vec{a}_y = 2E_0 \sin(\alpha x) \cos \omega t \vec{a}_y$

$$\vec{D} = 2\epsilon E_0 \sin(\alpha x) \cos \omega t \vec{a}_y \Rightarrow \frac{\partial \vec{D}}{\partial t} = -2\omega \epsilon E_0 \sin(\alpha x) \sin \omega t \vec{a}_y \quad \text{A/m}^2$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{D}}{\partial t} \Rightarrow \frac{\partial \vec{B}}{\partial t} = -2\alpha E_0 \cos \alpha x \cos \omega t \vec{a}_z$$

$$\text{Thus, } \vec{B} = -\frac{2\alpha}{\omega} E_0 \cos \alpha x \sin \omega t \vec{a}_z \quad \text{and } \vec{H} = -\frac{2\alpha}{\omega \mu} E_0 \cos \alpha x \sin \omega t \vec{a}_z \quad \text{A/m}$$

Problem 7.30  $\vec{H} = H_0 [\cos(\alpha x - \omega t) + \cos(\alpha x + \omega t)] \vec{a}_z$

$$= 2H_0 \cos(\alpha x) \cos \omega t \vec{a}_z \quad \text{A/m}$$

$$\frac{\partial \vec{D}}{\partial t} = \nabla \times \vec{H} = 2\alpha H_0 \sin(\alpha x) \cos \omega t \vec{a}_y \Rightarrow$$

$$\vec{D} = \frac{2\alpha}{\omega} H_0 \sin(\alpha x) \sin \omega t \vec{a}_y \quad \text{C/m}^2$$

$$\text{and } \vec{E} = \frac{2\alpha}{\omega \epsilon} H_0 \sin(\alpha x) \sin \omega t \vec{a}_y \quad \text{V/m}$$

Problem 7.31 Refer to the solution of Problem 7.29

$$\nabla \cdot \vec{B} = 0 \quad \text{and} \quad \nabla \cdot \vec{D} = 0$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \Rightarrow -\frac{\partial}{\partial x} H_z = \frac{\partial}{\partial t} (\epsilon E_y)$$

$$\text{or } \frac{\partial \alpha^2}{\omega \mu} E_0 \sin \alpha x \sin \omega t = \omega \epsilon E_0 \sin \alpha x \sin \omega t \Rightarrow \alpha^2 = \omega^2 \mu \epsilon$$

Problem 7.32 Refer to the solution of Problem 7.30

$$\nabla \cdot \vec{D} = 0 \quad \nabla \cdot \vec{B} = 0$$

$$\text{From } \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \text{ we have } \frac{\partial}{\partial x} E_y = -\frac{\partial}{\partial t} B_z \quad \text{or}$$

$$\frac{\partial \alpha^2}{\omega \epsilon} H_0 \cos \alpha x \sin \omega t = \omega \mu H_0 \cos \alpha x \sin \omega t \Rightarrow \alpha^2 = \omega^2 \mu \epsilon$$

Problem 7.33  $\vec{E} = E_0 \cos(\omega t - \beta z) \vec{a}_x$ ,  $\vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{D} = 0$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = -\beta E_0 \sin(\omega t - \beta z) \vec{a}_y \Rightarrow \vec{B} = \frac{\beta E_0}{\omega} \cos(\omega t - \beta z) \vec{a}_y$$

$$\vec{H} = \frac{\beta E_0}{\omega \mu} \cos(\omega t - \beta z) \vec{a}_y \quad \nabla \cdot \vec{B} = 0$$

$$\text{From } \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \text{ we obtain } \beta^2 = \omega^2 \mu \epsilon$$

$$w_e = \frac{1}{2} \vec{D} \cdot \vec{E} = \frac{1}{2} \epsilon E_0^2 \cos^2(\omega t - \beta z) \Rightarrow \langle w_e \rangle = \frac{1}{T} \int_0^T w_e dt = \frac{1}{4} \epsilon E_0^2 \text{ J/m}^3$$

$$w_m = \frac{1}{2} \vec{B} \cdot \vec{H} = \frac{1}{2} \mu H^2 = \frac{\beta^2}{2\omega^2 \mu} E_0^2 \cos^2(\omega t - \beta z) \Rightarrow \langle w_m \rangle = \frac{1}{T} \int_0^T w_m dt = \frac{1}{4} \frac{\beta^2}{\omega^2 \mu} E_0^2 = \frac{1}{4} \epsilon E_0^2$$

$$\langle \vec{S} \rangle = \frac{1}{T} \int_0^T (\vec{E} \times \vec{H}) dt = \frac{\beta}{\omega \mu} E_0^2 \frac{1}{T} \int_0^T \cos^2(\omega t - \beta z) dt \vec{a}_z = \frac{1}{2} \frac{\beta}{\omega \mu} E_0^2 \vec{a}_z \text{ W/m}^2$$

Problem 7.34  $\vec{E} = 2E_0 \sin \alpha x \cos \omega t \vec{a}_y$ ,  $\vec{H} = -\frac{\partial \alpha}{\omega \mu} E_0 \cos \alpha x \sin \omega t \vec{a}_z$

$$w_e = \frac{1}{2} \epsilon E^2 = 2\epsilon E_0^2 \sin^2 \alpha x \cos^2 \omega t \Rightarrow \langle w_e \rangle = \frac{1}{T} \int_0^T w_e dt = \epsilon E_0^2 \sin^2 \alpha x$$

$$w_m = \frac{1}{2} \mu H^2 = \frac{\partial \alpha^2}{\omega^2 \mu} E_0^2 \cos^2 \alpha x \sin^2 \omega t \Rightarrow \langle w_m \rangle = \frac{\alpha^2}{\omega^2 \mu} E_0^2 \cos^2 \alpha x \\ = \epsilon E_0^2 \cos^2 \alpha x \quad (\because \alpha^2 = \omega^2 \mu \epsilon) \\ [\text{Prob. 7.31}]$$

Problem 7.35  $\vec{E} = \frac{\partial \psi}{\partial t} H_0 \sin \alpha x \sin \omega t \vec{a}_y$

$\vec{H} = \partial H_0 \cos \alpha x \cos \omega t \vec{a}_z$

From Problem 7.30  
 $d^2 = \omega^2 \mu \epsilon$

$\omega_e = \frac{1}{\epsilon} \epsilon \vec{E} = \frac{\partial \omega^2}{\omega^2 \epsilon} H_0^2 \sin^2 \alpha x \sin^2 \omega t \Rightarrow \langle \omega_e \rangle = \mu H_0^2 \sin^2 \alpha x \quad \text{J/m}^3$

$\omega_m = \frac{1}{2} \mu \vec{H}^2 = \frac{1}{2} \mu H_0^2 \cos^2 \alpha x \cos^2 \omega t \Rightarrow \langle \omega_m \rangle = \mu H_0^2 \cos^2 \alpha x \quad \text{J/m}^3$

Problem 7.36  $q = \int i dt = \frac{I_m}{\omega} \sin \omega t$

$\oint \vec{D} \cdot d\vec{s} = q \Rightarrow D = \frac{q}{A} = \frac{I_m}{\omega A} \sin \omega t$

$J_d = \frac{\partial D}{\partial t} \Rightarrow J_d = \frac{I_m}{A} \cos \omega t$ , Thus,  $i_d = J_d A = I_m \cos \omega t$  A

Problem 7.37  $J_c = \sigma E$   $J_d = \omega \epsilon E \Rightarrow J_d/J_c = \frac{\omega \epsilon}{\sigma}$

For  $J_d/J_c = 1$ ,  $\omega \epsilon = \sigma \Rightarrow f = \frac{\sigma}{2\pi \epsilon}$

Sea water:  $\epsilon = 81 \epsilon_0$   $\sigma = 0.4 \times 10^{-3} \text{ S/m} \Rightarrow f = 88.889 \text{ kHz}$

when  $f \ll 88.889 \text{ kHz}$   $J_c \gg J_d$  conductor

and when  $f \gg 88.889 \text{ kHz}$ ,  $J_d \gg J_c$  poor conductor

Problem 7.38

$E_z = -\frac{V}{d} = \frac{141 \sin 10^9 t}{0.005} = -28.2 \sin 10^9 t \text{ kV/m}$

$\vec{J}_c = \sigma \vec{E} = -0.02 \times 28.2 \sin 10^9 t \vec{a}_z = -564 \sin 10^9 t \vec{a}_z \text{ A/m}^2$

$i_c = \int \vec{J}_c \cdot d\vec{s} = 564 \times 0.4 \sin 10^9 t = 225.6 \sin 10^9 t \text{ A}$

$\vec{J}_d = \epsilon \frac{\partial \vec{E}}{\partial t} = -\frac{4\pi \times 10^{-9}}{36\pi} \times 28.2 \times 10^9 \cos 10^9 t \vec{a}_z$

$= -997.37 \cos 10^9 t \vec{a}_z \text{ A/m}^2$

$i_d = \int \vec{J}_d \cdot d\vec{s} = 398.95 \cos 10^9 t \text{ A}$

$i = i_c + i_d = 225.6 \sin 10^9 t + 398.95 \cos 10^9 t \text{ A}$

$I_{rms} = \sqrt{\frac{225.6^2 + 398.95^2}{2}} = 324.08 \text{ A}$

Problem 7.39  $\vec{E} = E_0 \cos(\omega t - ax - kz) \vec{a}_y$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \frac{\partial B_x}{\partial t} = \frac{\partial E_y}{\partial z} = k E_0 \sin(\omega t - ax - kz) \Rightarrow$$

$$B_x = -\frac{k E_0}{\omega} \cos(\omega t - ax - kz)$$

and  $\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} = -a E_0 \sin(\omega t - ax - kz) \Rightarrow B_z = \frac{a E_0}{\omega} \cos(\omega t - ax - kz)$

$\nabla \cdot \vec{B} = 0$   $\nabla \cdot \vec{D} = 0$  Source-free:  $\vec{J} = 0$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \Rightarrow \nabla \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t} \Rightarrow$$

$$-(a^2 + k^2) \frac{E_0}{\omega} \sin(\omega t - ax - kz) \vec{a}_y = -\mu \epsilon \omega E_0 \sin(\omega t - ax - kz) \vec{a}_y$$

or  $\omega^2 \mu \epsilon = a^2 + k^2$  [condition for the fields to exist]

$$\omega_e = \frac{1}{2} \epsilon \vec{E}^2 = \frac{1}{2} \epsilon E_0^2 \cos^2(\omega t - ax - kz) \Rightarrow \langle \omega_e \rangle = \frac{1}{4} \epsilon E_0^2$$

$$\omega_m = \frac{1}{2} \frac{B^2}{\mu} = \frac{E_0^2}{2\mu} \left( \frac{a^2 + k^2}{\omega^2} \right) \cos^2(\omega t - ax - kz) \Rightarrow \langle \omega_m \rangle = \frac{E_0^2 (a^2 + k^2)}{4\mu \omega^2} = \frac{1}{4} \epsilon E_0^2$$

$$\vec{S} = \vec{E} \times \vec{H} = \frac{E_0^2}{\omega \mu} \cos^2(\omega t - ax - kz) [a \vec{a}_x + k \vec{a}_z]$$

$$\langle \vec{S} \rangle = \frac{E_0^2}{2\omega \mu} (a \vec{a}_x + k \vec{a}_z)$$

Problem 7.40  $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$  ①

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$
 ②

Take Curl of ①:  $\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H})$

Source-free:  $\nabla \cdot \vec{D} = 0$

$$\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Thus,  $\nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

Problem 7.41  $\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$  ①  $\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$  ②

Take Curl of ②:  $\nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} + \epsilon \frac{\partial}{\partial t} (\nabla \times \vec{E})$

$$\nabla (\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = -\mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad [\nabla \cdot \vec{B} = 0]$$

Thus,  $\nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$

or  $\nabla^2 \vec{H} - \mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$

Problem 7.42. For a dielectric Medium:

Source free:

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{D} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H}) \Rightarrow -\nabla^2 \vec{E} + \nabla (\nabla \cdot \vec{E}) = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\text{or } \nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla \times \nabla \times \vec{H} = \epsilon \frac{\partial}{\partial t} (\nabla \times \vec{E}) \Rightarrow \nabla (\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = -\mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

Problem 7.43

$$(\nabla \times \vec{E}) \cdot \vec{H}^* = -j\omega \vec{B} \cdot \vec{H}^* \quad (1) \quad (\nabla \times \vec{H}^*) \cdot \vec{E} = -j\omega \vec{D}^* \cdot \vec{E} \quad (2)$$

$$\text{However, } \nabla \cdot (\vec{E} \times \vec{H}^*) = (\nabla \times \vec{E}) \cdot \vec{H}^* - (\nabla \times \vec{H}^*) \cdot \vec{E}$$

Thus,

$$= -j\omega \vec{B} \cdot \vec{H}^* + j\omega \vec{D}^* \cdot \vec{E}$$

$$\text{Since } \hat{S} = \frac{1}{2} \vec{E} \times \vec{H}^*, \quad -\nabla \cdot \hat{S} = \frac{1}{2} j\omega [\vec{B} \cdot \vec{H}^* - \vec{E} \cdot \vec{D}^*] \\ = 2j\omega \left[ \frac{1}{4} \vec{B} \cdot \vec{H}^* - \frac{1}{4} \vec{E} \cdot \vec{D}^* \right]$$

Problem 7.44

For a conductive region:  $\vec{J} = \sigma \vec{E}$

$$\text{Thus, } \nabla \times \vec{E} = -j\omega \mu \vec{H} \quad \text{and} \quad \nabla \times \vec{H} = \sigma \vec{E} + j\omega \epsilon \vec{E}$$

$$(\nabla \times \vec{E}) \cdot \vec{H}^* = -j\omega \mu \vec{H} \cdot \vec{H}^* \quad (\nabla \times \vec{H}^*) \cdot \vec{E} = \sigma \vec{E} \cdot \vec{E}^* - j\omega \epsilon \vec{E} \cdot \vec{E}^*$$

$$\nabla \cdot (\vec{E} \times \vec{H}^*) = (\nabla \times \vec{E}) \cdot \vec{H}^* - (\nabla \times \vec{H}^*) \cdot \vec{E}$$

$$= -j\omega \mu \vec{H} \cdot \vec{H}^* - \sigma \vec{E} \cdot \vec{E}^* + j\omega \epsilon \vec{E} \cdot \vec{E}^*$$

$$\text{Since } \vec{H} \cdot \vec{H}^* = H^2 \quad \text{and} \quad \vec{E} \cdot \vec{E}^* = E^2,$$

$$\nabla \cdot (\vec{E} \times \vec{H}^*) = -\sigma E^2 - j\omega [\mu H^2 - \epsilon E^2]$$

$$\therefore \hat{S} = \frac{1}{2} (\vec{E} \times \vec{H}^*),$$

$$-\nabla \cdot \hat{S} = \frac{1}{2} \sigma E^2 + 2j\omega \left[ \frac{1}{4} \mu H^2 - \frac{1}{4} \epsilon E^2 \right]$$

Problem 7.45  $\tilde{E}_y = E_0 e^{-j\alpha x} e^{-jkz} \Rightarrow \tilde{D}_y = \epsilon E_0 e^{-j\alpha x} e^{-jkz} \Rightarrow \nabla \cdot \tilde{D} = 0$

$$\nabla \times \tilde{E} = - \frac{\partial \tilde{E}_y}{\partial z} \vec{a}_x + \frac{\partial \tilde{E}_y}{\partial x} \vec{a}_z = j(k \vec{a}_x - \alpha \vec{a}_z) e^{-j\alpha x} e^{-jkz} E_0$$

Since  $\nabla \times \tilde{E} = -j\omega\mu \tilde{H} \Rightarrow \tilde{H} = \left( \frac{\alpha E_0}{\omega\mu} \vec{a}_z - \frac{k E_0}{\omega\mu} \vec{a}_x \right) e^{-j\alpha x} e^{-jkz} \quad A/m$

$\nabla \cdot \tilde{D} = 0 \Rightarrow \nabla \cdot \tilde{H} = 0$  and  $\nabla \cdot \tilde{H} = \frac{\partial \tilde{H}_x}{\partial x} + \frac{\partial \tilde{H}_z}{\partial z} = + \frac{j k E_0 \alpha}{\omega\mu} - j \frac{\alpha E_0 k}{\omega\mu} = 0$

$$\nabla \times \tilde{H} = j \frac{E_0}{\omega\mu} [k^2 + \alpha^2] e^{-j\alpha x} e^{-jkz} \vec{a}_y$$

From  $\nabla \times \tilde{H} = j\omega\epsilon \tilde{E}$ , we obtain  $k^2 + \alpha^2 = \omega^2\mu\epsilon$  as the condition for fields to exist.

$$\langle W_e \rangle = \frac{1}{4} \tilde{E} \cdot \tilde{D}^* = \frac{1}{4} \epsilon E_0^2 \quad J/m^3$$

$$\langle W_m \rangle = \frac{1}{4} \tilde{B} \cdot \tilde{H}^* = \frac{1}{4} \mu H^2 = \frac{k^2 + \alpha^2}{4\omega^2\mu} E_0^2 = \frac{1}{4} \epsilon E_0^2 \quad J/m^3$$

$$\langle \hat{S} \rangle = \frac{1}{2} \operatorname{Re} [\tilde{E} \times \tilde{H}^*] = \frac{E_0^2}{2\omega\mu} [k \vec{a}_z + \alpha \vec{a}_x] \quad W/m^2$$

Problem 7.46  $\tilde{E}_\rho = \frac{V}{\rho \ln(b/a)} e^{-jkz} \quad V/m, \quad \tilde{H}_\phi = \frac{I}{2\pi\rho} e^{-jkz} \quad A/m$

$$\nabla \times \tilde{E} = \frac{\partial \tilde{E}_\rho}{\partial z} \vec{a}_\phi = -j \frac{kV}{\rho \ln(b/a)} e^{-jkz} \vec{a}_\phi$$

Since  $\nabla \times \tilde{E} = -j\omega\mu_0 \tilde{H} \Rightarrow -j \frac{kV}{\rho \ln(b/a)} e^{-jkz} = -j\omega\mu_0 \frac{I}{2\pi\rho} e^{-jkz}$ , or  $k = \frac{\omega\mu_0 I}{2\pi V} \ln(b/a)$  ①

$$\nabla \times \tilde{H} = \frac{1}{\rho} \left( -\frac{\partial}{\partial z} \rho \tilde{H}_\phi \right) \vec{a}_\rho = \frac{j k I}{2\pi\rho} e^{-jkz} \vec{a}_\rho$$

Since  $\nabla \times \tilde{H} = j\omega\epsilon_0 \tilde{E} \Rightarrow k = \frac{2\pi V \omega\epsilon_0}{I \ln(b/a)}$  ②

From ① and ②:  $k^2 = \omega^2\mu_0\epsilon_0$

$$\langle \hat{S} \rangle = \frac{1}{2} \operatorname{Re} [\tilde{E} \times \tilde{H}^*] = \frac{VI}{4\pi\rho^2 \ln(b/a)} \vec{a}_z$$

Finally,  $\langle P \rangle = \int_S \langle \hat{S} \rangle \cdot d\vec{s} = \frac{VI}{4\pi \ln(b/a)} \int_a^b \frac{1}{\rho} d\rho \int_0^{2\pi} d\phi = \frac{1}{2} VI \quad W$

Note that  $V$  and  $I$  are the max. values.

Problem 7.47  $\vec{A} = \vec{A}_r + j\vec{A}_i \Rightarrow \vec{A}(t) = \vec{A}_r \cos \omega t - \vec{A}_i \sin \omega t$

$\vec{B} = \vec{B}_r + j\vec{B}_i \Rightarrow \vec{B}(t) = \vec{B}_r \cos \omega t - \vec{B}_i \sin \omega t$

$\vec{A} \cdot \vec{B}^* = \vec{A}_r \cdot \vec{B}_r + \vec{A}_i \cdot \vec{B}_i - j(\vec{A}_r \cdot \vec{B}_i - \vec{A}_i \cdot \vec{B}_r) \quad \textcircled{1}$

$\vec{A} \cdot \vec{B} = \vec{A}_r \cdot \vec{B}_r \cos^2 \omega t + \vec{A}_i \cdot \vec{B}_i \sin^2 \omega t - (\vec{A}_r \cdot \vec{B}_i + \vec{A}_i \cdot \vec{B}_r) \sin \omega t \cos \omega t$

Average:  $\langle \vec{A} \cdot \vec{B} \rangle = \frac{1}{T} \int_0^T (\vec{A} \cdot \vec{B}) dt = \frac{1}{2} (\vec{A}_r \cdot \vec{B}_r + \vec{A}_i \cdot \vec{B}_i) \quad \textcircled{2}$

From ① and ②:  $\langle \vec{A} \cdot \vec{B} \rangle = \frac{1}{2} \text{Re} [\vec{A} \cdot \vec{B}^*]$

Problem 7.48 Using the definitions given in Problem 7.47

$\vec{A} \times \vec{B}^* = \vec{A}_r \times \vec{B}_r + \vec{A}_i \times \vec{B}_i + j(\vec{A}_i \times \vec{B}_r - \vec{A}_r \times \vec{B}_i) \quad \textcircled{1}$

$\vec{A} \times \vec{B} = (\vec{A}_r \times \vec{B}_r) \cos^2 \omega t + (\vec{A}_i \times \vec{B}_i) \sin^2 \omega t - (\vec{A}_r \times \vec{B}_i + \vec{A}_i \times \vec{B}_r) \sin \omega t \cos \omega t$

$\langle \vec{A} \times \vec{B} \rangle = \frac{1}{2\pi} \int_0^{2\pi} (\vec{A} \times \vec{B}) d\omega t = \frac{1}{2} [\vec{A}_r \times \vec{B}_r] + \frac{1}{2} [\vec{A}_i \times \vec{B}_i] \quad \textcircled{2}$

Comparing ① and ②:  $\langle \vec{A} \times \vec{B} \rangle = \frac{1}{2} \text{Re} [\vec{A} \times \vec{B}^*]$

Problem 7.49  $\nabla \times \vec{E} = -j\omega\mu \vec{H} \quad \nabla \times \vec{H} = (\sigma + j\omega\epsilon) \vec{E}$

$\nabla \times \nabla \times \vec{E} = -j\omega\mu \nabla \times \vec{H} \Rightarrow \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -j\omega\mu(\sigma + j\omega\epsilon) \vec{E}$

Source free:  $\vec{P}_v = 0 \Rightarrow \nabla \cdot \vec{D} = 0$  or  $\nabla \cdot \vec{E} = 0$

Hence:

$\nabla^2 \vec{E} + \omega^2 \mu \epsilon \vec{E} - j\omega\mu\sigma \vec{E} = 0$

Problem 7.50

$\nabla \times \vec{E} = -j\omega\mu \vec{H}$

$\nabla \times \vec{H} = \sigma \vec{E} + j\omega\epsilon \vec{E}$

$\nabla \times \nabla \times \vec{H} = (\sigma + j\omega\epsilon) \nabla \times \vec{E} \Rightarrow \nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = -j\omega\mu(\sigma + j\omega\epsilon) \vec{H}$

Since  $\nabla \cdot \vec{B} = 0 \Rightarrow \nabla \cdot \vec{H} = 0$

Hence:

$\nabla^2 \vec{H} = j\omega\mu(\sigma + j\omega\epsilon) \vec{H}$

or  $\nabla^2 \vec{H} + \omega^2 \mu \epsilon \vec{H} - j\omega\mu\sigma \vec{H} = 0$

Problem 7.51  $\vec{E}_z = 1000 e^{-j\beta x} \text{ V/m}$   $\vec{H}_y = -\frac{1000}{\eta} e^{-j\beta x} \text{ A/m}$   $\beta = \frac{\pi}{3} \text{ rad/m}$

$\nabla \times \vec{E} = j\beta 1000 e^{-j\beta x} \vec{a}_y$ , Since  $\nabla \times \vec{E} = -j\omega\mu_0 \vec{H} \Rightarrow \eta = \frac{\omega\mu_0}{\beta}$  or  $\beta\eta = \omega\mu_0$  ①

$\nabla \cdot \vec{D} = 0$   $\nabla \cdot \vec{B} = \nabla \cdot \vec{H} = 0$   $\epsilon = 4\epsilon_0$

$\nabla \times \vec{H} = j \frac{1000}{\eta} \beta e^{-j\beta x} \vec{a}_z$ , Since  $\nabla \times \vec{H} = j\omega\epsilon \vec{E} \Rightarrow \frac{\beta}{\eta} = 4\omega\epsilon$  ②

From ① and ②:  $\beta^2 = \omega^2 4\mu_0\epsilon \Rightarrow \omega = \frac{\beta}{2\sqrt{\mu_0\epsilon}} = 0.5\pi \times 10^8 \text{ rad/s}$

$\omega = 2\pi f \Rightarrow f = 25 \text{ MHz}$  and  $\eta = \frac{\omega\mu_0}{\beta} = 60\pi \Omega$

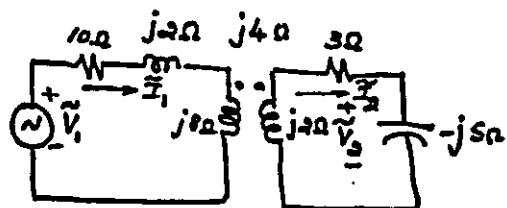
$\langle \hat{S} \rangle = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] = \frac{1000^2}{\eta} \vec{a}_x \text{ W/m}^2$   $0.54$

$\langle P \rangle = \int \langle \hat{S} \rangle \cdot d\vec{s} = \frac{1000^2}{\eta} \int_0^4 dy \int_0^4 dz = 21.22 \text{ kW}$

Problem 7.52  $\omega = 1000 \text{ rad/s}$

$j\omega L_1 = j8\Omega$ ,  $j\omega L_2 = j2\Omega$   $-\frac{j}{\omega C} = -j5\Omega$

$j\omega M = \sqrt{j\omega L_1 \times j\omega L_2} = j4\Omega$   $\tilde{V}_1 = 120 \angle 0^\circ \text{ V}$



$$\begin{cases} (10 + j10)\tilde{I}_1 - j4\tilde{I}_2 = 120 \\ -j4\tilde{I}_1 + (3 - j3)\tilde{I}_2 = 0 \end{cases} \Rightarrow \begin{cases} \tilde{I}_1 = 6.699 \angle -45^\circ \text{ A} \\ \tilde{I}_2 = 6.316 \angle 90^\circ \text{ A} \end{cases}$$

$\tilde{V}_2 = [(-j5)(6.316 \angle 90^\circ)]$   
 $= 31.58 \angle 0^\circ \text{ V}$

Problem 7.53  $\tilde{V}_1 = 120 \angle 30^\circ \text{ V}$  (rms)

$\tilde{I}_1 = \frac{\tilde{V}_1}{Z} = 5.303 \angle 75^\circ \text{ A}$

$P_m = \text{Re}[\tilde{V}_1 \tilde{I}_1^*] = 450 \text{ W}$

$\tilde{I}_2 = a_1 \tilde{I}_1 = 2.6515 \angle 75^\circ \text{ A}$

$\tilde{I}_3 = a_2 \tilde{I}_2 = 10.616 \angle 75^\circ \text{ A}$

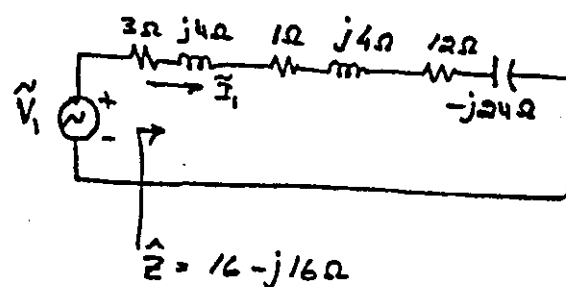
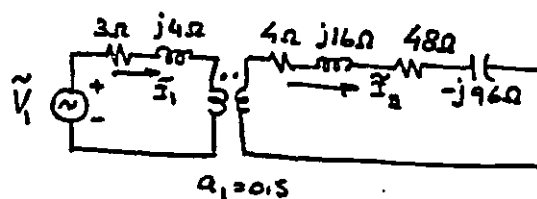
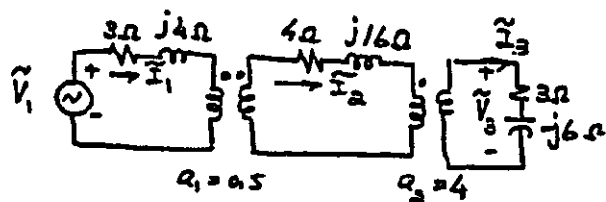
Load voltage:  $\tilde{V}_3 = \tilde{I}_3 [3 - j6\Omega]$   
 $= 71.214 \angle 11.57^\circ \text{ V}$

Power to load:  $P_L = 10.616^2 \times 3 = 338 \text{ W}$

$\eta = \frac{P_L}{P_{in}} = 0.75$  or 75%

Power lost on Transmission Line:

$P_{TL} = 2.6515^2 \times 4 = 28.12 \text{ W}$





### Problem 7.54

$$a = 30/750 = 0.04 \quad N_1 = 30 \quad N_2 = 750 \quad f = 50 \text{ Hz}$$

$$\text{pf} = 0.8 \text{ lag} \Rightarrow \theta = -36.87^\circ. \text{ Thus, } \tilde{I}_2 = 4 \angle -36.87^\circ \text{ A}, \quad \tilde{I}_1 = \frac{\tilde{I}_2}{a} = 100 \angle -36.87^\circ \text{ A}$$

$$\text{From } E = 4.44 f N_1 \Phi_m \Rightarrow \Phi_m = \frac{240}{4.44 \times 50 \times 30} \approx 36 \text{ mWb. (Peak)}$$

### Problem 7.55

$$\Phi_m = \sqrt{2} \Phi_{rms} = 1.414 \text{ mWb (max value)}$$

$$\text{From } E = 4.44 f N \Phi_m, \Rightarrow N = \frac{230 \times 10^3}{4.44 \times 60 \times 1.414} \approx 610 \text{ Turns}$$

### Problem 7.56

$$I_2 = \frac{1000}{120} = 8.33 \text{ A} \quad \text{pf} = 0.6 \text{ lead} \Rightarrow \tilde{I}_2 = 8.333 \angle 53.13^\circ \text{ A}$$

$$\text{Thus, } \hat{Z}_L = \frac{120}{\tilde{I}_2} = 14.4 \angle -53.13^\circ = 8.64 - j11.52 \Omega$$

### Problem 7.57

Two-winding Transformer:  $V_1 = 120 \text{ V}, V_2 = 480 \text{ V}, S = 480 \text{ VA}$

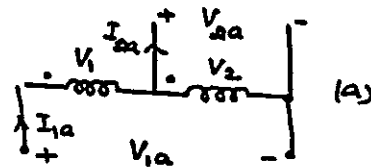
a)

$$V_{1a} = 600 \text{ V}, \quad V_{2a} = 480 \text{ V}$$

$$I_1 = 4 \text{ A}, \quad I_2 = 10 \text{ A} \quad a = 0.25$$

$$I_{1a} = 4 \text{ A}, \quad I_{2a} = 4 \times \frac{600}{480} = 5 \text{ A}$$

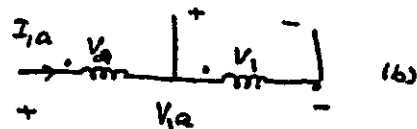
$$S = 480 \times 5 = 24 \text{ kVA}$$



$$b) \quad V_{1a} = 600 \text{ V} \quad V_{2a} = 120 \text{ V}$$

$$I_{1a} = 10 \text{ A} \quad I_{2a} = 10 \times \frac{600}{120} = 50 \text{ A}$$

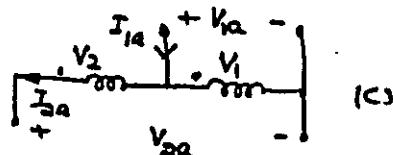
$$S = 120 \times 50 = 6 \text{ kVA}$$



$$c) \quad V_{1a} = 120 \text{ V} \quad V_{2a} = 600 \text{ V}$$

$$I_{2a} = 10 \text{ A} \quad I_{1a} = 10 \times \frac{600}{120} = 50 \text{ A}$$

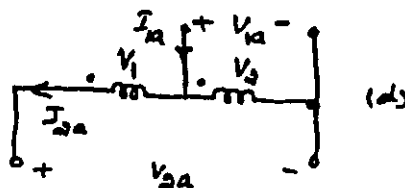
$$S = 600 \times 10 = 6 \text{ kVA}$$



$$d) \quad V_{1a} = 480 \text{ V} \quad V_{2a} = 600 \text{ V}$$

$$I_{2a} = 4 \text{ A} \quad I_{1a} = 5 \text{ A}$$

$$S = 600 \times 4 = 24 \text{ kVA}$$



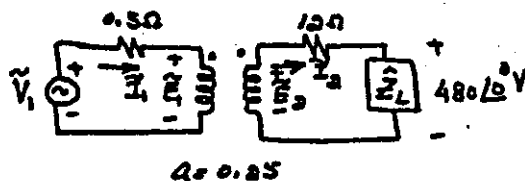
Problem 7.58  $\tilde{I}_2 = 10 \angle -53.13^\circ \text{ A}$ ,  $\tilde{I}_1 = 40 \angle -53.13^\circ \text{ A}$

$$P_o = 480 \times 10 \times 0.6 = 2880 \text{ W}$$

$$\tilde{E}_2 = 480 + 12 \times 10 \angle -53.13^\circ = 552 - j96 \text{ V}$$

$$\tilde{E}_1 = a \tilde{E}_2 = 138 - j24 \text{ V}, \quad \tilde{V}_1 = \tilde{E}_1 + 0.5 \tilde{I}_1 = 150 - j40 = 155.242 \angle -14.93^\circ \text{ V}$$

$$P_{in} = R[\tilde{V}_1 \tilde{I}_1^*] = R[155.242 \times 40 \angle (53.13^\circ - 14.93^\circ)] = 4880 \text{ W} \quad \eta = \frac{P_o}{P_{in}} = 0.59 \approx 59\%$$



Problem 7.59

(a)

$$\tilde{I}_{1a} = \tilde{I}_1 = 40 \angle -53.13^\circ \text{ A}$$

$$a = \frac{120}{480} = 0.25$$

$$Q_T = 600/480 = 1.25$$

$$\tilde{I}_{2a} = Q_T \tilde{I}_{1a} = 50 \angle -53.13^\circ \text{ A}$$

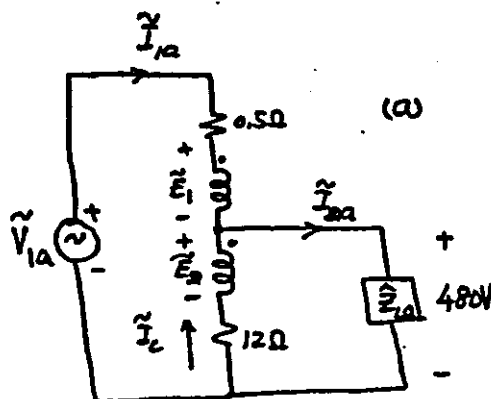
$$\tilde{I}_c = \tilde{I}_{2a} - \tilde{I}_{1a} = 10 \angle -53.13^\circ \text{ A}$$

$$\tilde{E}_2 = 480 + \tilde{I}_c (12) = 552 - j96 \text{ V}$$

$$\tilde{E}_1 = a \tilde{E}_2 = 138 - j24 \text{ V}$$

$$\tilde{V}_{1a} = 0.5 \tilde{I}_{1a} + \tilde{E}_1 + 480 = 630 - j40 = 631.27 \angle -3.63^\circ \text{ V}$$

$$P_o = R[\tilde{V}_2 \tilde{I}_{2a}^*] = 14,400 \text{ W} \quad P_{in} = R[\tilde{V}_{1a} \tilde{I}_{1a}^*] = 16399 \text{ W} \Rightarrow \eta = \frac{P_o}{P_{in}} = 0.878 \approx 87.8\%$$



(b)  $\tilde{I}_{1a} = 10 \angle -53.13^\circ \text{ A}$ ,  $Q_T = 600/120 = 5$

$$\tilde{I}_{2a} = 50 \angle -53.13^\circ \text{ A}, \quad \tilde{I}_c = 40 \angle -53.13^\circ \text{ A}$$

$$\tilde{E}_1 = 120 + 0.5 \tilde{I}_c = 132 - j16 \text{ V}$$

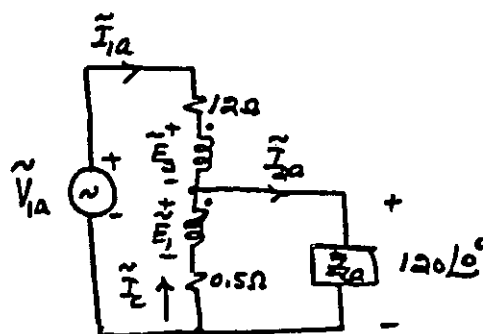
$$\tilde{E}_2 = 528 - j64 \text{ V}$$

$$\tilde{V}_{1a} = 12 \tilde{I}_{1a} + \tilde{E}_2 + 120 = 737.56 \angle -12.53^\circ \text{ V}$$

$$P_o = R[120 \tilde{I}_{2a}^*] = 3600 \text{ W}$$

$$P_{in} = R[\tilde{V}_{1a} \tilde{I}_{1a}^*] = 5600 \text{ W}$$

$$\eta = \frac{P_o}{P_{in}} = 0.643 \approx 64.3\%$$



Another Method:

$$P_{in} = P_o + P_{cu}$$

$$P_{cu} = I_c^2 (0.5) + I_{1a}^2 (12)$$

$$= 0.5 \times 40^2 + 12 \times 10^2 = 2000 \text{ W}$$

$$\text{Thus, } P_{in} = 3600 + 2000 = 5600 \text{ W}$$

$$c) \tilde{I}_{2a} = 40 \angle -53.13^\circ \text{ A} \quad a_T = 40/600 = 0.8$$

$$\tilde{I}_{1a} = 50 \angle -53.13^\circ \text{ A}, \quad \tilde{I}_c = 10 \angle -53.13^\circ$$

$$\tilde{E}_1 + \tilde{E}_2 = 600 + 0.5 \tilde{I}_{2a} - 12 \tilde{I}_c$$

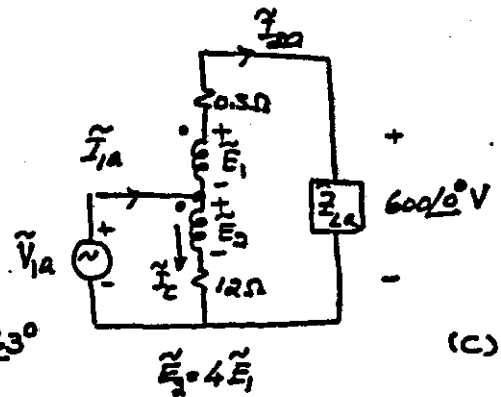
$$5\tilde{E}_1 = 540 + j80 \quad \text{or} \quad \tilde{E}_1 = 108 + j16$$

$$\tilde{V}_{1a} = 12\tilde{I}_c + \tilde{E}_2 = 12\tilde{I}_c + 4\tilde{E}_1 = 505.015 \angle -3.63^\circ$$

$$P_o = R_L [600 \times 40 \angle -53.13^\circ] = 14,400 \text{ W}$$

$$P_{in} = R_L [\tilde{V}_{1a} \tilde{I}_{1a}^*] = 16,399 \text{ W}$$

$$\eta = \frac{P_o}{P_{in}} = 0.878 \quad \text{or} \quad 87.8\%$$



(c)

$$d) \tilde{I}_{2a} = 10 \angle -53.13^\circ \text{ A} \quad a_T = 120/600 = 0.2$$

$$\tilde{I}_{1a} = \frac{\tilde{I}_{2a}}{a_T} = 50 \angle -53.13^\circ \text{ A}$$

$$\tilde{I}_c = 40 \angle -53.13^\circ \text{ A}$$

$$\tilde{E}_1 + \tilde{E}_2 = 600 + 12 \tilde{I}_{2a} - 0.5 \tilde{I}_c = 660 - j80$$

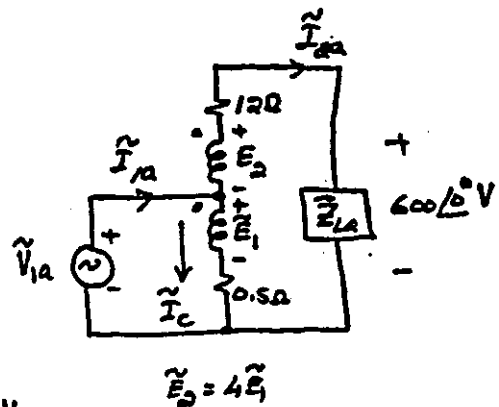
$$\tilde{E}_1 = 132 - j16$$

$$\tilde{V}_{1a} = \tilde{E}_1 + 0.5 \tilde{I}_c = 144 - j32 = 147.51 \angle -12.53^\circ \text{ V}$$

$$P_o = R_L [600 \tilde{I}_{2a}^*] = 3600 \text{ W}$$

$$P_{in} = R_L [\tilde{V}_{1a} \tilde{I}_{1a}^*] = 5200 \text{ W}$$

$$\eta = \frac{P_o}{P_{in}} = 0.643 \quad \text{or} \quad 64.3\%$$



### Problem 7.60

$$\Phi = 1.5 \sin(100\pi t) \text{ wb} \quad a = 0.75 \text{ m}$$

$$\vec{F} = -e\vec{E} = 1.6 \times 10^{-17} \cos(100\pi t) \hat{a}_\phi$$

$$\vec{B} = \frac{\Phi}{\pi a^2} \hat{a}_2 = 0.8488 \sin(100\pi t) \hat{a}_2 \text{ T}$$

$$W = 2\pi a F = 7.54 \times 10^{-17} \cos(100\pi t) \text{ J}$$

$$\text{Thus, } B_m = 0.8488 \text{ T}$$

$$\langle W \rangle = \frac{2}{\pi} W = 4.8 \times 10^{-17} \text{ J} = 300 \text{ eV}$$

$$\vec{E} = -a\omega B_m \cos(100\pi t) \hat{a}_\phi$$

Number of Trips:

$$= -100 \cos(100\pi t) \hat{a}_\phi$$

$$N = \frac{90 \times 10^6}{300} = 300,000$$